Optimal basis and optical gain matrix for pyramid sensor in the visible

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Optical Gain generalization

Modal OG:

- rely on calibration procedure $A_{r_0} = \langle(\mathbf{P}'(\phi_{res})m_i)\rangle_{ens.}$ and reconstructor
- return a scalar value per mode $\rightarrow$ modal OG vector (off-diagonals?..)
- are very suitable for modal control

Zonal OG ?

- Literature: zonal reconstructor discarded, zonal OG is noise-alike, unusable ...
- Linz: good performance with zonal MMSE reconstructor in R band ...

Path forward:

- How does the OG in our zonal basis look like?
- OG = OG (REC details):

  mmse/map, regularization, calibration amplitudes, wavefront representation basis basis (e.g., zonal – DM IFs or artificial), inversion method, mode filtering, etc.

Optical Gain Matrix

- Assume $Z = \{z_j\}, M = \{m_j\}$ are complete, invertible, span the same spaces,

$$M = ZB.$$

- Due to linearity of calibration and averaging

$$A_{mod}(0) = A_{zon}(0)B, \qquad \langle A_{mod}(res)\rangle = \langle A_{zon}(res)\rangle B. \tag{1}$$

- Define zonal $R_{zon} : s \to \phi_z$ and modal $R_{mod} : s \to \phi_z$ WF reconstructors as

$$A^{-1} := (A^T A + \alpha C^{-1})^{-1} A^T, \tag{2}$$

$$R_{zon} = A_{zon}^{-1}, \qquad R_{mod} = BA_{mod}^{-1}. \tag{3}$$

- Theory vs practice (condition number)

$$\boxed{R_{zon} = R_{mod}.} \tag{4}$$

Optical Gain Matrix (cont.)

- Define $OG\ matrix = OG\ matrix(basis, R, r_0, \lambda)$

$$G_Z := \langle A_{zon}(res)\rangle^{-1} A_{zon}(0), \qquad G_M := \langle A_{mod}(res)\rangle^{-1} A_{mod}(0). \tag{5}$$

- Using same information (= calibration), but not using the norms
- By construction, G_Z is well-defined

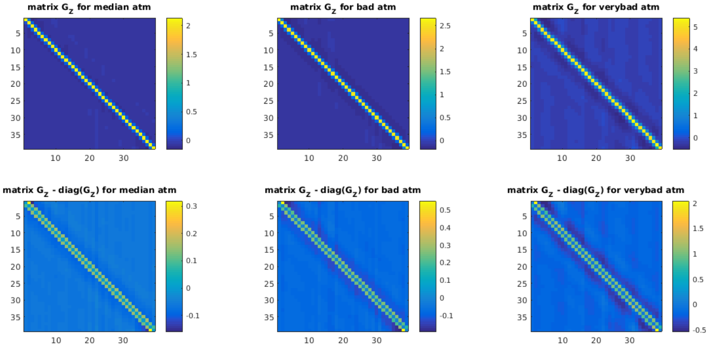
$$G_Z B = B G_M. \tag{6}$$

- Theorem:
  modal and zonal OG corrections are equivalent and can be applied with any (modal or zonal) reconstructor,

$$OGC_{zon}(R_{zon}) = OGC_{mod}(R_{mod}). \tag{7}$$

Zonal OGM

- G_Z has a clear structure (not noise!)



- Purity, diagonal approximation $\rightarrow$ scalar gain for zonal basis, boundary effects
- $G_Z = G_Z(zonal\ basis\ type, \ldots)$
  (virtual basis is Linz' decoupled approach, DM IFs otherwise)

Numerical results: half-ELT setting

	Latency 2 frames	K-band Strehl
(1)	Modal MAP + scalar OG [2]	60,0
(2)	Modal REC + scalar OG [1]	60,9
(3)	P-CuReD + scalar OG [2]	64,2
(4)	Modal REC + modal OGV [1]	64,6
(5)	Zonal MMSE + scalar OG [2]	65,2
(6)	Modal REC + full modal OGM	tbd
(7)	Zonal MMSE + full zonal OGM	tbd
(8)	Houdini (direct projection) [2]	74,0

Half-ELT setting	
telescope diameter D	19m
central obstruction	no
spiders	no
sensing wavelength	658nm
frame rate	500 Hz
modulation	4 lambda/D
number of subapertures	38
number of actuators	39
r0 at 500nm	0,09m
subaperture size d	0,5m
latency, frames	2 and 3
guide star magnitude	13
photons/subap/frame	40

- diagonal approximation (scalar in zonal case)
- (4,5): diagonal zonal OGM approximation is good! uniform sensor response to all virtual IFs
- (8) Houdini gives a best estimate (a non-linear reconstructor could achieve)
- (6-7): using full zonal/modal OGM will improve zonal/modal REC results
- (3) Model-based (calibration-free) REC is good!

[1] Results from COMPASS kindly provided by V. Deo

[2] Results obtained in OCTOPUS

Thanks

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