Efficient model-based wavefront control* using nonlinear optimization

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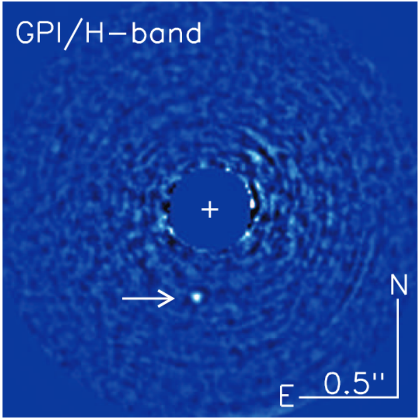
*(for coronagraphy)

The search for life on extrasolar planets

- More than 4000 exoplanets have been discovered since 1992
  - Mostly via transit timing, radial velocities (indirect)
- Future direct imaging missions (HabEx[1], LUVOIR[2]) will seek to detect and characterize Earth-like planets around Sun-like stars
- Requires instruments capable of suppressing starlight by factor of 10^{10} at small angular separations (< 1 arcsec)



Source: Macintosh et al., Science **350**(6256), 64–67 (2015)

[1] https://www.jpl.nasa.gov/habex/
[2] https://asd.gsfc.nasa.gov/luvoir/

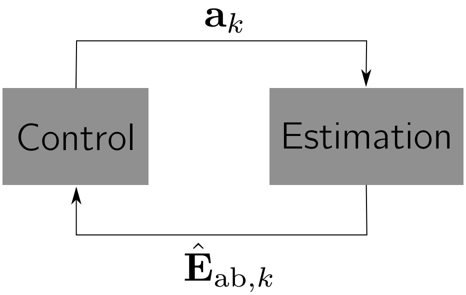
Focal-plane wavefront sensing and control

Aberrated E-field → $\mathbf{E}_{\mathrm{ab},k}$; E-field from DMs → $\mathbf{E}_{\mathrm{DM},k}$

$$\mathbf{E}_{D,k} \approx \mathbf{E}_{\mathrm{ab},k} + \mathbf{E}_{\mathrm{DM},k}(\mathbf{a}_k)$$

Total E-field at detector → $\mathbf{E}_{D,k}$; Iteration index → k; DM actuator commands → $\mathbf{a}_k$

- **Quasi-static speckle** from diffracted starlight dominates dark zone
- **Focal-plane wavefront estimation** eliminates non-common path aberrations
- **Closed-loop wavefront control** iteratively minimizes quasi-static speckle
- **Two deformable mirrors** (one in-pupil and one out-of-pupil) correct amplitude and phase aberrations over symmetric dark zone



Stroke minimization[3]

$$\underset{\mathbf{a}_k}{\arg\min} \quad \mathbf{a}_k^T \mathbf{a}_k \quad \text{subject to} \quad \overbrace{\mathbf{E}_{D,k}^\dagger \mathbf{E}_{D,k}}^{\text{Integrated dark-zone intensity}} \leq \underbrace{I_{T,k}}_{\text{Target}} \tag{1}$$

- Method of Lagrange multipliers: find critical point of Lagrangian function

$$\mathcal{L}_k \triangleq \mathbf{a}_k^T \mathbf{a}_k + \mu(\mathbf{E}_{D,k}^\dagger \mathbf{E}_{D,k} - I_{T,k}) \tag{2}$$

- **Conventional approach**:

1. Use numerical model to evaluate **Jacobian matrix** $\mathbf{G}_k \triangleq \dfrac{\partial \mathbf{E}_{D,k}}{\partial \mathbf{a}_k}$. Then

$$\mathbf{a}_k^*(\mu) = -\left[\frac{1}{\mu} + \Re\left\{\mathbf{G}_k^\dagger \mathbf{G}_k\right\}\right]^{-1} \Re\left\{\mathbf{G}_k^\dagger \widehat{\mathbf{E}}_{\mathrm{ab},k}\right\} \tag{3}$$

2. Generate family of solutions with different μ; choose smallest μ such that $\mathbf{E}_{D,k}^\dagger \mathbf{E}_{D,k} \leq I_{T,k}$

[3] L. Pueyo et al., *Appl. Opt.* **48**, 6296–6312 (2009)

Stroke minimization

$$\mathbf{a}_k^*(\mu) = -\left[\frac{1}{\mu} + \Re\left\{\mathbf{G}_k^\dagger \mathbf{G}_k\right\}\right]^{-1} \Re\left\{\mathbf{G}_k^\dagger \widehat{\mathbf{E}}_{\mathrm{ab},k}\right\} \tag{4}$$

- **Problem**: have to calculate matrix-valued derivative $\mathbf{G}_k \in \mathbb{C}^{N_{\mathrm{pix}} \times N_{\mathrm{act}}}$
  - **Computationally expensive**
  - Usually just compute $\mathbf{G}_0$ before start of experiment and reuse for all iterations $\rightarrow$ reduces speed of convergence to high contrast
  - For multi-wavelength control, need to compute separate $\mathbf{G}$ matrix for each controlled wavelength
- Can we do better?

The solution: reverse-mode algorithmic differentiation (RMAD)[5]

- Efficient, analytical differentiation of **numerical algorithms**
- **Main idea**: given **forward model** consisting of sequence of N differentiable operations $x_n = f_n(x_{n-1})$ with x_N scalar, construct **adjoint model** that evaluates $\partial x_N / \partial x_n \triangleq \overline{x}_n$
  - Adjoint model can be constructed manually (by programmer) or automatically (by computer)
- Cost of evaluating gradient $\sim$ cost of evaluating forward model (**cheap gradient principle**[4])

[4] A. S. Jurling and J. R. Fienup, *J. Opt. Soc. Am. A* **31**(7), 1348–59 (2014)

[5] Also known as **backpropagation algorithm** in machine learning with neural networks

Proposed algorithm[6]

- Stroke minimization, but with quadratic contrast penalty:

$$J_k = \mathbf{a}_k^T \mathbf{a}_k + \mu \left(\mathbf{E}_{D,k}^{\dagger} \mathbf{E}_{D,k} - I_{T,k} \right)^2 \tag{5}$$

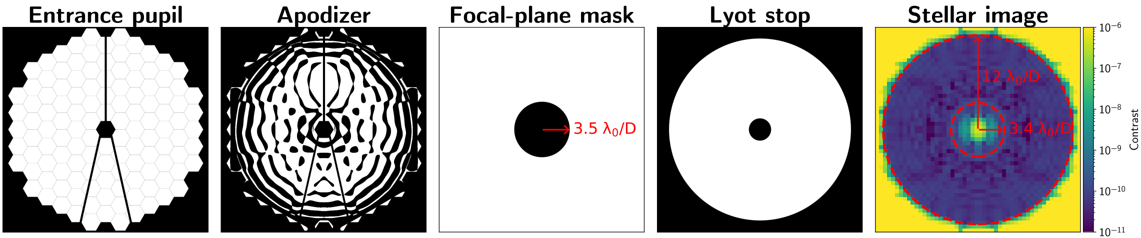
  - Quadratic penalty enforces contrast target without line search on μ

- Compute actuator solutions via gradient-based nonlinear optimization of J_k with RMAD gradient $\bar{\mathbf{a}}_k$
- **Advantages**:
  - Don't need to calculate $\mathbf{G}_k$ at all $\rightarrow$ { Massive reduction in up-front computation; Easier to update model between iterations }
  - Only **vector-valued derivatives** handled during gradient computation $\rightarrow$ better scaling with actuator count, dark zone size

[6] S. D. Will, T. D. Groff, and J. R. Fienup, *JATIS* (2020, under review)

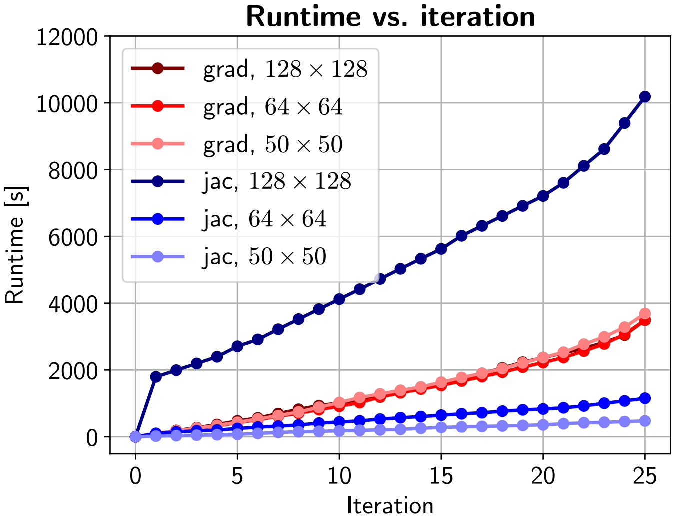
Simulations: overview

- Compared proposed gradient-based method to conventional Jacobian-based algorithm
- Small-angle APLC design[7] submitted to 2020 Astrophysics Decadal Survey for proposed LUVOIR mission[8]
- Three different MEMS DM formats: 50×50, 64×64, 128×128



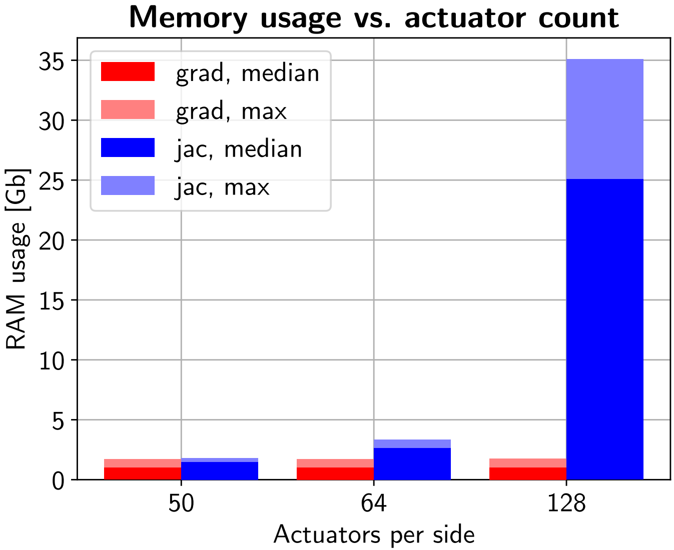
[7] Courtesy of R. Soummer
[8] https://asd.gsfc.nasa.gov/luvoir/

Simulation results



- Both algorithms converge to 10^{-10} contrast after 25 iterations
- Jacobian-based algorithm faster for lowest actuator counts, but very slow for 128×128 case
- Proposed algorithm runtime invariant to actuator count
- Results shown do **not** factor in time cost of precomputing $\mathbf{G}_0$

Simulation results



- Memory consumption of proposed algorithm invariant to actuator count
- At 128×128 actuators per DM (baseline for LUVOIR mission), Jacobian-based algorithm consumes $10\times$ more memory

Future work

- Laboratory demonstrations
- **Dynamically updated model**: capture time-varying effects in system using e.g. data from low-order wavefront sensor (LOWFS)
- **Adaptive control**:
  - Compute gradients with respect to model parameters, use control history to tune model
    - Influence function, pupil shear, pupil transmittance, etc.
  - Alternate between contrast improvement and model improvement

The Jacobian matrix (for in-pupil DM)

$$\mathbf{G}_{1,k} \triangleq \left[\cdots \quad i\frac{4\pi}{\lambda}\mathcal{C}\left\{\mathbf{P} \circ \exp\left\{i\phi_{1,k-1}\right\} \circ \mathbf{f}_{1,j};\lambda\right\} \quad \cdots\right]$$

Pupil transmittance → $\mathbf{P}$

DM1 phase (previous iteration) → $\phi_{1,k-1}$

Coronagraph operator → $\mathcal{C}$

jth influence function → $\mathbf{f}_{1,j}$

Wavelength → λ

- $\mathbf{G}_{1,k}$ is a function of previous DM commands $\Rightarrow$ should be recomputed in each control iteration
- $\mathcal{C}\{\cdot\}$ captures *a priori* knowledge of coronagraph behavior